Determination of the Admittance of a General Waveguide-Coaxial Line Junction

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Abstract —An analytical approximate method to determine the admittance of a general coaxial-waveguide junction is presented. The accuracy of the method is verified experimentally for the case of rectangular waveguide.

I. INTRODUCTION

This paper presents a straightforward means for determining the admittance Y measured at the coaxial aperture of a junction between a coaxial line of characteristic impedance Z_c and a general microwave or millimeter-wave guiding structure. The problem is of considerable interest in the design of solid-state oscillators in which the device is mounted in a coaxial line coupled to a waveguide [1], [2]. The junction to be analyzed is shown in Fig. 1, the guiding structure is of a general class which includes rectangular waveguide, shielded image guide, H-guide, and groove guide.

Studies previously reported on waveguide-coaxial line junctions have used an empirical factor [3], slowly-convergent Hankel-function series [4], or numerical integration [5]. By contrast, the approach presented here yields good accuracy while using simple trigonometric functions.

II. APPROACH

An analysis recently completed by the present authors [5] derived two expressions for the admittance Y defined above. The basic features of this analysis are as follows.

1) The coaxial aperture is replaced by a surface magnetic current $\overline{M}$, located on a conductor closing the aperture, and calculated using the TEM-mode approximation.

2) Extending the outer coaxial conductor into the guide gives a coaxial cavity excited by $\overline{M}$. The resulting TEM field in this cavity and the associated currents flowing on the inner and outer conductors are easily found by use of transmission-line theory.

3) To recover the original structure, the outer conductor of the line is removed from the guide and is replaced by a current which is the negative of that found in 2) (using the Schelkunoff principle [5]). The current produces a field in the guide, determined by a modal approach.

4) The original field is a superposition of the two fields from Steps 2) and 3) above. The field in the guide can also be considered as a superposition of TEM and TM radial modes. To find admittance of the junction, the extraction of the TEM coaxial mode at the junction plane is required; this is accomplished by using the relations which hold for the axial electric field and for the ϕ-component of magnetic field for radial modes, and by means of orthogonality between the modes.

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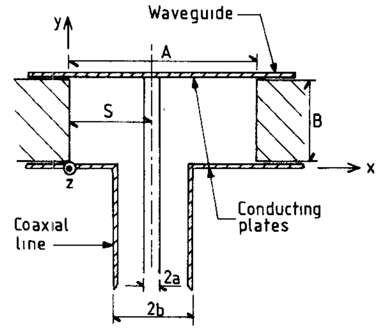


Fig. 1 Coaxial-line—waveguide junction.

The approach presented here replaces the circular contours in the line integrals of these expressions by square contours, greatly simplifying the computation while retaining good accuracy.

Taking TEM-mode fields in the coaxial line, and representing the field in the waveguide by a set of modes with fields $\overline{E}_{mn}$ and $\overline{H}_{mn}$, and propagation constant Γ_{mn}, we define a set of coefficients $F_{ij}(m,n)$ to describe coupling between the fields of the m,nth mode and that at the coaxial aperture S_a, where

$$F_{ij}(m,n)=\frac{1}{2l_il_jP_{mn}}\int_{c_i}\int_{c_j}E_{ymn}(x,y=0)\cdot E_{ymn}(x',y'=0)\,e^{-\Gamma_{mn}|z-z'|}\,dl\,dl' \qquad (1)$$

with

$$P_{mn}=\int_{S_w}\left(\overline{E}_{mn}\times\overline{H}_{mn}\right)\bar{a}_z\,dS,$$

$$-\Gamma_{mn}^2=k^2-k_{xm}^2-k_{yn}^2;$$

k_{xm}, k_{yn}—eigenvalues in the x and y directions.

S_w is the waveguide cross-sectional surface. The circular contours of the inner and outer coaxial conductors are represented by C_1 and C_2, and the contour lengths by l_1 and l_2.

For the considered class of waveguides $E_{ymn}(x,y=0)$ is assumed to be in a form of trigonometric functions $\begin{cases}\sin k_{xm}x\\ \cos k_{xm}x\end{cases}$. Using the fact that the fields in the extended coaxial region within the waveguide can be expressed in terms of radial-line modes, Y can be shown [5] to be

$$Y=-\frac{j\cot kB}{Z_c}+\sum_{n=0}^{\infty}\left[\frac{k}{Z_c\left(k^2-k_{yn}^2\right)}\right]^2 D_n \qquad (2)$$

where k is the wave number, $k_{yn}=n\pi/B$, and

$$D_n=\left[\sum_m F_{12}(m,n)\right]^2\Big/\sum_m F_{11}(m,n)-\sum_m F_{22}(m,n).$$

An alternative approach rests upon determination of the current $I(y)$ on the inner coaxial conductor. Using coaxial-line theory, together with a general dyadic Green's-function expansion, we obtain [5]

$$I(y)=-j\frac{V}{Z_c}\frac{\cos k(y-B)}{\sin kB}+\sum_{n=0}^{\infty}\frac{A_n\epsilon_{0n}}{B}\cos k_{yn}y \qquad (3)$$

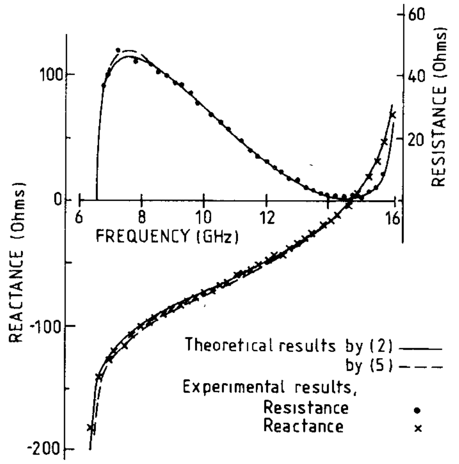


Fig. 2 Comparison between theoretical and experimental values of input impedance at the coaxial aperture plane, for rectangular guide with $A=22.86$ mm, $B=10.16$ mm, $S=A/2$, and with $a=1.55$ mm, $b=3.55$ mm for 50-Ω line.

with

$$A_n=\frac{jkV}{\left(k^2-k_{yn}^2\right)Z_c}\sum_m F_{12}(m,n)\Big/\sum_m F_{11}(m,n)$$

and where ϵ_{on} is the Neumann factor and V is the voltage across the aperture S_a. The admittance can be expressed as

$$Y=\frac{I(y=0)}{V}-\frac{jk}{2\pi VZ_c}\int_{S_a}E_y\ln\left(\frac{b}{r}\right)dS. \quad (4)$$

To good approximation we can assume that the real part of E_y/V changes linearly with the distance $r-a$ from the inner conductor, and the imaginary part varies as a quadratic function, and we can also approximate $\ln(b/r)/\ln(b/a)$ by $(b-r)/(b-a)$ to obtain

$$Y=\frac{I(y=0)}{V}+\frac{l_2^2-l_1^2}{3\pi l_2}\sum_{n=0}^{\infty}\frac{k^2\ln\left(\frac{b}{a}\right)G_n}{Z_c^2\left(k^2-k_{yn}^2\right)} \quad (5)$$

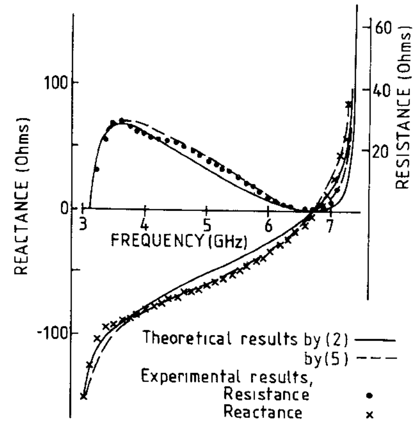
where

$$G_n=(l_1+l_2)\operatorname{Re}(D_n)+(0.4l_1+0.6l_2)\operatorname{Im}(D_n).$$

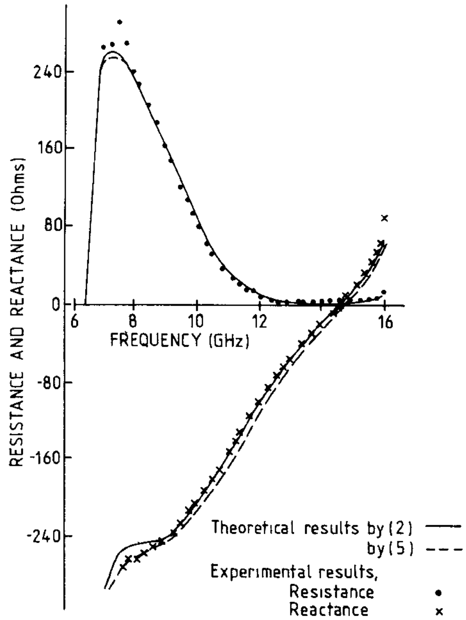
The form given by (4) has been found convenient [5] for small apertures, where the integral term can be neglected. Equation (5) is useful for small and medium-size apertures, while (2) should be accurate for a wide range of aperture sizes.

It is evident that evaluation of all 3 expressions for Y depends upon determination of the coupling coefficients $F_{ij}(m,n)$ through calculation of the line integrals along C_1 and C_2. If the circular contours are retained, these integrals give rise to a slowly convergent series of Hankel-function terms in the expression for Y; this requires a large computer and extended computational time.

The approach used here is to replace the circular contours by square contours W_1, W_2 such that W_1 has the same perimeter length as C_1, and W_2 gives the same aperture area as is given by C_2, i.e., the inner contour is of side-length $2a_1=1.57a$, and the outer contour of side-length $2b_1=1.77b\ [1-0.215(a/b)^2]^{1/2}$. Using W_1 and W_2 with the square sides parallel to the x, z



(a)



(b)

Fig 3.(a) Comparison between theoretical and experimental values of input impedance at the coaxial aperture plane for a rectangular guide with $A=47.6$ mm, $B=22.15$ mm, and $S=A/2$; for 24.5-Ω line with $a=2.37$ mm and $b=3.55$ mm. (b) Comparison between theoretical and experimental values of input impedance at the coaxial aperture plane for a rectangular guide with $A=22.86$ mm, $B=10.16$ mm, and $S=A/2$; for 133-Ω line with $a=0.76$ mm and $b=7.15$ mm.

directions and $E_{ymn}(x, y=0)=\sin k_{xm}x$, we can readily show

$$F_{12}=\frac{\sin^2 k_{xm}S}{16P_{mn}k_{xm}\Gamma_{mn}a_1b_1}$$
$$\cdot\left[\frac{\Gamma_{mn}}{k_{xm}}(E_-+E_+)\sin k_{xm}a_1\sin k_{xm}b_1\right.$$
$$+(E_--E_+)\sin k_{xm}b_1\cos k_{xm}a_1$$
$$+(2-E_--E_+)\sin k_{xm}a_1\cos k_{xm}b_1$$
$$\left.+\frac{k_{xm}}{\Gamma_{mn}}(2a_1\Gamma_{mn}+E_+-E_-)\cos k_{xm}a_1\cos k_{xm}b_1\right] \quad (6)$$

where

$$E_+ = e^{-\Gamma_{mn}(b_1 + a_1)}$$

$$E_- = e^{-\Gamma_{mn}(b_1 - a_1)}$$

and similar expressions are found for F_{11} and F_{22}. It is clear that these expressions are rapidly evaluated without recourse to Bessel or Hankel function computation.

III. Accuracy of Approximation

Calculations of impedance were carried out for a junction between a coaxial line and rectangular waveguide, using the square contour approximation, and the results compared with previously published experimental measurements [3]. The infinite series was truncated at $m = 3A/2a$ and $n \leqslant 10$.

Fig. 2 shows the comparison for $Z_c = 50\ \Omega$, using (2) and (5), both of which give good accuracy. For $Z_c < 50\ \Omega$, Fig. 3(a) shows (5) gives good accuracy, but (2) shows some error; when $Z_c > 50\ \Omega$, Fig. 3(b) shows that both (2) and (5) give good accuracy.

Overall, the comparison with experimental measurement shows that (5) yields good accuracy, with the square-contour approximation, and that it provides a useful alternative to use of the circular contours for the coaxial-line cross-sectional boundaries.

IV. Conclusion

Although the approach developed here has been applied only to rectangular waveguide junctions, it is evident from the formulation that it will have useful application in a wide range of waveguides having conducting planes on their upper and lower surfaces. The key element in the analysis is derivation of a dyadic Green's function for the waveguide, accomplished readily if a complete set of modes can be found for that guiding structure; note that only one component of the dyadic Green's function is required in this analysis.

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Coupled Microstrips on Double Anisotropic Layers

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***Abstract*—A variational expression is presented for the mode capacitances of coupled microstrip lines with double anisotropic layered substrates. It is shown that the even- and odd-mode phase velocities can be made equal when a sapphire layer is deposited on a boron nitride substrate. High coupler directivity can be achieved with appropriate values of both layers' thicknesses.**

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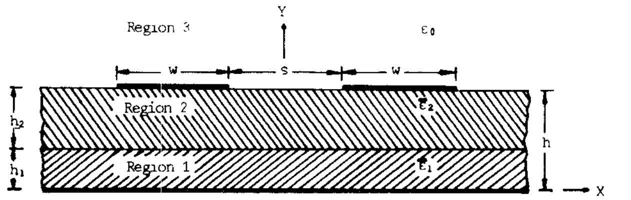

Fig. 1 Cross section of coupled microstrips on double anisotropic substrate layers.

I. Introduction

The analysis of structures such as single [1]–[4] and coupled [5]–[11] microstrip lines, slot lines [12], and coplanar waveguides [13] on anisotropic substrates, has received considerable attention in the last few years as a consequence of the interesting properties of crystalline substrate materials, such as sapphire [1] and boron nitride, or some glass- and ceramic-filled polymeric materials, e.g. Duroid and Epsilam, which are electrically anisotropic substances [10].

Certain types of anisotropy can be advantageous for coupled microstrip components. Alexopoulos and Krowne show in [7] that the difference between the odd- and even-phase velocities will be reduced if the permittivity in the direction parallel to the ground plane is greater than the perpendicular tensor component, in an anisotropic substrate. To equalize the phase velocities, the parallel component must be approximately twice the perpendicular component. This anisotropy is nonexistent in practical microwave materials [10]. Kobayashi and Terakado [8] propose to equalize the phase velocities by rotating the crystal axes an angle θ with respect to the microstrip axes. Recently, Alexopoulos and Maas [10], [14] have demonstrated that high coupler directivity can be achieved by making use of the substrate anisotropy in conjunction with a top cover to equalize even- and odd-mode phase velocities.

In this paper, coupled microstrip lines in which the substrate is made with two layers of anisotropic dielectrics are studied. An accurate expression to compute the quasi-static characteristic parameters of this structure has been obtained.

A layer of sapphire on another of boron nitride has been used as a composite substrate to equalize the even- and odd-mode phase velocities. Therefore, the coupler directivity can be improved by making use of this substrate with appropriate values of both layers' thicknesses.

II. Analysis Method

Fig. 1 shows the coupled microstrip lines to be analyzed. The substrate is composed of two layers of anisotropic dielectric (region 1 and 2).

The anisotropy is described by two permittivity tensors $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$, respectively, which are given by

$$\bar{\epsilon}_i = \epsilon_0 \begin{bmatrix} \epsilon_i^{11} & \epsilon_i^{12} \\ \epsilon_i^{12} & \epsilon_i^{22} \end{bmatrix} \qquad i = 1,2. \tag{1}$$

It is assumed that the strips are infinitely thin and that the conductors are perfect.

In order to compute the characteristic parameters of this structure, we have extended, in this paper, the quasi-static approach which was used in [4], [9], [11] to obtain a variational expression for the modal capacitances.

To this end, the solution of the Laplace's equation

$$\vec{\nabla} \cdot (\bar{\epsilon}_i \cdot \vec{\nabla} \psi_i) = 0 \tag{2}$$